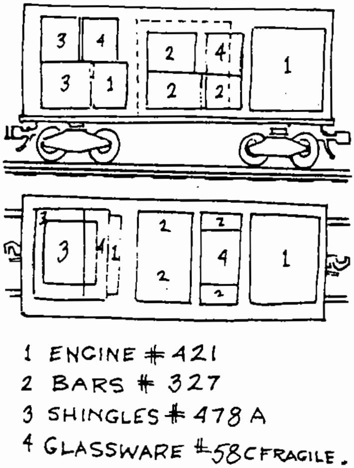

How Accountants Save Money By Sampling

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Accountancy and statistics are regarded by many people as two of the dullest subjects on earth. The essays in this volume, it is hoped, will change such views about statistics. This essay deals with important uses of statistics in accounting practice, and it may also reveal some interesting facets of accounting.

All of us, after all, want to use our money efficiently and effectively. We shall see how the use of statistical sampling in accounting saves money for railroads and airlines as they face problems of dividing revenues among several carriers. Similar statistical sampling methods are used in other areas of accounting and auditing work. Indeed, they are used in many fields of business, government, and science.

Accountants and auditors traditionally have insisted on accuracy in the accounting records of firms and other organizations. This insistence has led them to do much work on a complete, 100% basis. For instance, auditors may want to check the value of the inventory that a firm has on hand. To do this, they may examine the entire inventory; that is, they may actually count how many units of each type of inventory item are on hand, determine the value of each kind of unit, and thus finally obtain the total value of the inventory.

As another instance, an auditor may want to know the proportion of accounts receivable that have been owed for 60 or more days. This information may be needed to verify a reserve for bad debts. Accounts that have not been paid within 60 days are more susceptible to bad debt losses than accounts that have been open a shorter time. In order to establish the proportion of accounts receivable that are 60 days old or older, the auditor may examine every single account receivable held by the firm and determine for each the amount of money owed for 60 or more days.

Is it necessary to conduct these 100% examinations of inventory, of accounts receivable, or of similar collections, in order to obtain the figures that the accountant needs? More specifically, could a sample adequately provide the information needed by the accountant without all of the tedious work necessary for a complete, 100% examination? Let us focus on the inventory items.

In statistical terminology, the group of inventory items for which the total value is to be ascertained is called the *population* of interest. A *sample* selected from such a population consists of some, but not all, of the items in the population. A sample is selected to find out about characteristics of the population without looking at every element of the population.

The cost of examining a relatively small sample of inventory items is usually less than the cost of a complete examination because the sample requires an examination of fewer items. But are the results based on the relatively small sample almost as good as those from a complete examination?

Experience with sampling in many areas has shown that relatively small samples frequently provide results that are almost as good as those obtained from a complete examination, while at the same time the sample results cost considerably less. Indeed, sometimes the sampling results are even better than those from a 100% examination. That statement may seem startling, but consider the task of taking an inventory in a large company. Many persons are required for the task. Because of the size of the undertaking, it may be hard to give thorough training to these persons, and the quality check on the work may have to be limited. On the other hand, a small sample of the inventory items would require fewer persons, and therefore they could be trained better. Furthermore, the quality control program for the inventory could be more rigorous when a smaller number of persons are involved. The net effect might well be that the sample results are more accurate than the 100% enumeration! That is, the gains in accuracy from better training and quality control with a small sample may more than balance the sampling error introduced by selecting only a sample of inventory items instead of all of them. Of course, the sampling must be done intelligently and properly. The study of sampling is an important part of statistics.

THE CHESAPEAKE AND OHIO FREIGHT STUDY

Statements that relatively small samples can provide results almost as good as those from a complete examination, or indeed sometimes even better, are often not convincing by themselves. Statisticians have therefore often found it helpful to conduct studies that compare the results of a sample with those of a complete enumeration. Such a study was made by the Chesapeake and Ohio Railroad Company in determining the amount of revenue due them on interline, less-than-carload freight shipments. If a shipment travels over several railroads, the total freight charge is divided among them. The computations necessary to determine each railroad's revenue are cumbersome and expensive. Hence the Chesapeake and Ohio experimented to determine if the division of total revenue among several railroads could be made accurately on the basis of a sample and at a substantial saving in clerical expense.

In one of these experiments, they studied the division of revenue for all less-than-carload freight shipments traveling over the Pere Marquette district of the Chesapeake and Ohio and another railroad (to be called A for confidentiality), during a six-month period. The waybills of these shipments constituted the population under examination. A waybill, a document issued with every shipment of freight, gives details about the goods, route, and charges. From it, the amounts due each railroad can be computed. The total number of waybills in the population was known, as well as the total freight revenue accounted for by the population of waybills. The problem was to determine how much of this total revenue belonged to the Chesapeake and Ohio.

For the six-month perod under study, there were nearly 23,000 waybills in the population. Since the amounts of the freight charges on these waybills vary greatly (some freight charges were as low as $2, others as high as $200), it was decided to use a sampling procedure called *stratified sampling*. With this procedure, the waybills in the population are first divided into relatively homogeneous subgroups called *strata*. The subgroups in this instance were set up according to the amount of the total freight charge, since the amount due the Chesapeake and Ohio on a waybill tended to be related to the total amount of the waybill. That is, the larger the total amount of a waybill, the larger tended to be the amount due the Chesapeake and Ohio on that waybill. Specifically, the strata were as follows:

Stratum	*Waybills with Charges Between*
1	$ 0 and $ 5.00
2	$ 5.01 and $10.00
3	$10.01 and $20.00
4	$20.01 and $40.00
5	$40.01 and over

Note that each stratum contains waybills with total freight charges of roughly the same order of magnitude. Because of the general tendency by which the amount due the Chesapeake and Ohio varied with the total freight charge on a waybill, each stratum is relatively more homogeneous with respect to the amount of freight charges due the Chesapeake and Ohio. At the same time, the strata differ substantially from one another.

Statistical theory then was used to decide how large a sample from each stratum must be selected so that the amount of the revenue due the Chesapeake and Ohio could be estimated with required precision from as small a sample as possible. One piece of information needed for this determination was the number of waybills in each stratum. The sampling rates decided on for the strata were:

Stratum	*Proportion to Be Sampled*
1	1%
2	10%
3	20%
4	50%
5	100%

Note that this theory led to larger sampling rates in the strata containing wider ranges of freight charges and smaller sampling rates in the strata containing narrow ranges of freight charges. To understand this, consider stratum 1, containing waybills with charges between $0 and $5.00. Here the variation between the waybill amounts is small, and therefore a small sample will provide adequate information about the amounts of all of the waybills in that stratum. On the other hand, stratum 4, containing waybills with charges between $20.01 and $40.00, has much greater variation. A larger sample is therefore required in this stratum to obtain adequate information about the amounts of all waybills in that stratum. In an unreal extreme situation where all the waybills in a stratum would have the same amount due the Chesapeake and Ohio, a sample of just one waybill would provide all the information about the waybill amounts in that stratum.

Once the sample sizes were determined, the next problem was to select the samples from each stratum. For a statistician to be able to evaluate the precision of the sample results (that is, how close the sample results are likely to be to the relevant population characteristic), the sample must be selected according to a known probability mechanism. Various methods of probability sampling are available. One is called ***simple random sampling***. This type of sample may be directly selected by use of a random number generator or by use of a table of random numbers, a portion of which is illustrated in Table 1. How might Table 1 be used to select a simple random sample from each of the strata? Consider stratum 1 and suppose it contains 9,000 waybills, which we label with four-digit numbers from 0001 to 9000. We want to obtain four-digit numbers from the table; we might start in the upper left-hand corner, using columns 1 through 4. The first number obtained is 1328. Our first sample waybill is then the one numbered 1,328. Our second sample waybill would be 2,122. The next number from the table of random digits is 9905, but there

Table 1 Portion of a table of random digits

Line	*(1)–(5)*	*(6)–(10)*	*(11)–(15)*	*(16)–(20)*	*(21)–(25)*	*(26)–(30)*	*(31)–(35)*
101	13284	16834	74151	92027	24670	36665	00770
102	21224	00370	30420	03883	94648	89428	41583
103	99052	47887	81085	64933	66279	80432	65793
104	00199	50993	98603	38452	87890	94624	69721
105	60578	06483	28733	37867	07936	98710	98539
106	91240	18312	17441	01929	18163	69201	31211
107	97458	14229	12063	59611	32249	90466	33216
108	35249	38646	34475	72417	60514	69257	12489
109	38980	46600	11759	11900	46743	27860	77940
110	10750	52745	38749	87365	58959	53731	89295
111	36247	27850	73958	20673	37800	63835	71051
112	70994	66986	99744	72438	01174	42159	11392
113	99638	94702	11463	18148	81386	80431	90628
114	72055	15774	43857	99805	10419	76939	25993
115	24038	65541	85788	55835	38835	59399	13790
116	74976	14631	35908	28221	39470	91548	12854
117	35553	71628	70189	26436	63407	91178	90348
118	35676	12797	51434	82976	42010	26344	92920
119	74815	67523	72985	23183	02446	63594	98924
120	45246	88048	65173	50989	91060	89894	36036

Source: Table of 105,000 Random Decimal Digits. Interstate Commerce Commission, Bureau of Transport Economics and Statistics, Washington, D.C., May 1949.

are only 9,000 waybills in the stratum, so we pass over this number and go on to the next one, which is 0019. This process would be continued until the required sample of 90 (1% of 9,000) has been obtained. The digits in the table of random digits are generated so that all numbers (four-digit numbers in our case) are equally likely.

Another method of selecting waybills from each stratum is called *serial number sampling,* and this was the method actually used by the Chesapeake and Ohio Railroad. In this procedure, the sample within each stratum is selected according to certain digits in the serial number of the waybill. In this particular case, the last two digits in the serial number of the waybill were used. To explain how these last two digits are used to select the sample, consider stratum 1, with its 1% sample. The number of possible pairs of digits appearing in the last two places of the serial number (00, 01, 02, . . . , 99) is 100. If one of these pairs is chosen from a table of random digits and all waybills with these last two digits in their serial number are selected for the sample, it will be found that about 1% of the stratum is included in the sample. For stratum 1, the random number turned out to be 02. Therefore all waybills with freight charges of $5 or less whose last two serial number digits were 02 were selected for the sample. The serial number digits used for the other strata, as well as the sampling rates, were as follows:

Stratum	*Proportion to be Sampled*	*Waybills with Numbers Ending in:*
1	1%	02
2	10%	2
3	20%	2 or 4
4	50%	00 to 49
5	100%	00 to 99

Since the serial numbers appear prominently on the waybills, this procedure is a simple one for selecting the sample. Furthermore, in this case experience indicates that it provides essentially the equivalent of a simple random sample from each stratum.

Altogether, 2,072 waybills out of 22,984 in the population (9%) were chosen according to this procedure. For each waybill in the sample, the amount of freight revenue due the Chesapeake and Ohio was calculated. For each stratum, the total amount due for the population of waybills was then estimated, and these estimates were added to obtain an estimate of the total amount of freight revenue due the Chesapeake and Ohio on the almost 23,000 waybills in the population. Because this was an experiment, a complete examination of the population was also made, so that the sample result could be compared with the result obtained from an analysis of all waybills in the population. The findings were:

Total amount due Chesapeake and Ohio on basis of complete examination of population	$64,651
Total amount due Chesapeake and Ohio on basis of sample	64,568
Difference	$ 83

Thus a sample of only about 9% of the waybills provided an estimate of the total revenue due the Chesapeake and Ohio within $83 of the figure obtained from a complete examination of all waybills. Because the sample cost no more than $1,000, while the complete examination cost about $5,000, the advantages of sampling are apparent. It just does not make sense to spend $4,000 to catch an error of $83. Furthermore, although the error in this instance was against the Chesapeake and Ohio, the next time it may be against another railroad, so that the long run cumulative error is relatively even smaller.

OTHER RAILROAD AND AIRLINE SAMPLING STUDIES

The Chesapeake and Ohio conducted the same type of test for interline passenger receipts. They studied tickets sold during a five-month period to commercial passengers traveling only on the Chesapeake district of the Chesapeake and Ohio and on two other railroads, A and B. The findings are shown in Table 2. Again, these results dramatically demonstrate the ability of relatively small samples to provide precise estimates of the total revenue due the Chesapeake and Ohio.

Airlines also have used statistical sampling to estimate their share of the revenue on tickets for passengers traveling on two or more airlines. Three airlines tested statistical sampling during a four-month period. In that time, the degree of error in the sample estimate based on relatively small samples did not exceed 0.07% (that is, $700 in $1,000,000) for any of the three airlines. On the basis of this experiment, wider use of statistical sampling in settling interline accounts has been made. At one point in time, the sample consisted of about 12% of the interline tickets and the cumulative sampling error was running at less than 0.1%. The clerical savings were estimated to be near $75,000 annually for some of the larger carriers and more than $500,000 for the industry.

Statistical sampling in accounting and auditing has also been used to estimate the value of inventory on hand, the proportion of accounts receivable balances

Table 2 Results of passenger ticket study

	100% Examination	*5% Sample*	*Difference Dollars*	*Difference Percent*
Railroad A				
Total number of tickets	14,109			
Total revenue	$325,600			
Chesapeake and Ohio portion of total revenue	$212,164	$212,063	− $101	−0.05
Railroad B				
Total number of tickets	7,652			
Total revenue	$128,503			
Chesapeake and Ohio portion of total revenue	$ 79,710	$ 80,057	+ $347	+0.44

Source: Railway Age, June 9, 1952.

that are 60 days old or older, and the proportion of accounts receivable balances that are acknowledged as correct by the customer. In each instance, it has been demonstrated that a relatively small sample, carefully drawn and examined, can furnish results that are of high quality and at a much lower cost than with a complete examination.

SAMPLING FOR RADIO ROYALTIES

Another area where statistical sampling is used for accounting purposes determines the distribution of royalties to composers and publishers for music played on the radio. A performing rights organization, BMI, collects fees that entitle radio stations to use BMI's affiliates' music. Each station pays a blanket fee, proportional to the revenue of the station. These fees then must be apportioned appropriately to the composers and publishers of the music actually played. Information on the music played on the radio stations cannot be obtained on a 100% census basis because it would be much too costly. There are around 9,000 stations playing music an average of 18 hours per day. Thus, for 365 days in a year, these stations play approximately 59 million hours of music.

Instead, BMI selects a sample of radio stations to determine what music is being played. It does so by first stratifying all of the stations in the United States according to urbanity, region of the country, and amount of music played per day. This is done by a computerized program that also groups the stations according to their similarity with respect to the proportion of BMI music that they play. This stratification is made annually and usually leads to eight or nine strata. Each stratum is then divided into two substrata according to the size of the fee paid to BMI, so that stations that pay small fees and stations that pay large fees are placed into different strata. In addition to these strata, another stratum is established for new stations that go into operation during the year.

A sample of radio stations is then randomly selected each quarter from each stratum. For each selected station, two or three days during the quarter are randomly chosen, with the days distributed uniformly over the quarter. During these days, each selected station keeps a log of every song played. These logs are then returned to BMI, the information is entered into the computer, and the music is identified either as music for which BMI is the representative or music for which BMI is not the representative. From this sample information, projections are made of the amount of payments that BMI should make to each music publisher and composer each quarter. Over the course of a year all radio stations are asked to log one specified period.

SUMMARY

To summarize, statistical sampling consists of the selection of a number of items from a population, with the selection done in such a way that every possible sample from the population has a known probability of being chosen. Frequently, a statistical sample can provide reliable information at much lower cost than a complete examination. Also, a statistical sample often can provide more timely data than a complete enumeration of the population because fewer data have to be collected and smaller amounts of data need to be processed. Finally, a statistical sample can sometimes provide more accurate information than a complete enumeration when quality control over the data collection can be carried on more effectively on a small scale.

PROBLEMS

1. Explain the difference between simple random sampling and serial number sampling.
2. Suppose a university administrator is considering ordering some new desks for classrooms. She needs to find out how many desks already in use need to be replaced.
   a. Should she consider using sampling methods in this situation? What are the arguments for sampling? Against?
   b. If she did use sampling methods, what would the *population* be?
3. Why was stratified sampling used in the C&O freight study?
4. Refer to Table 2. Add the Railroad A and Railroad B ticket revenues, and find the difference in percent between a 5% sample and a 100% examination.
5. In the C&O freight study, how large a percentage of the total amount due C&O was the result of error due to sampling?
6. An army psychologist wants to take a sample of 1,000 enlisted men to find out their attitudes toward the "new Army." He obtains a list of the names of 10,000 enlisted men arranged by squads; each squad has 10 men, with a sergeant heading the list, then a corporal, followed by 8 privates.
   a. Would you recommend that the psychologist use serial number sampling (using the digits 0–9) to choose a sample of 1,000 from this list of 10,000 men? Why?
   b. If the psychologist used serial number sampling, what would be the chance of getting only sergeants in his sample? What if he used simple random sampling?
   c. Answer the questions in (a) and (b) if the psychologist used a list that placed the 10,000 enlisted men in alphabetical order.
7. Suppose C&O and Railroad A sampled tickets to determine their share of revenue from interline passenger receipts every month for a year. For how many months would you expect the sampling error to favor C&O?
8. Use Table 1 to draw a random sample of 25 two-digit numbers. How many are even? How many have both digits even? Do the same for a random sample of 100 two-digit numbers. Compare your answers to those obtained by others. What conclusions can you draw?

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